Feedback Control of Turbulent Wall Flows

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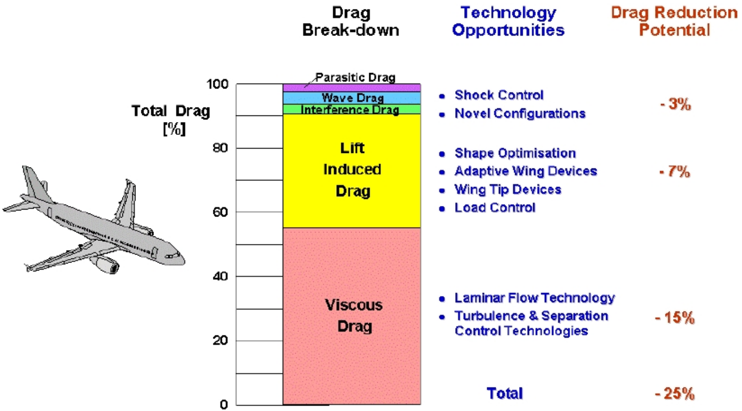
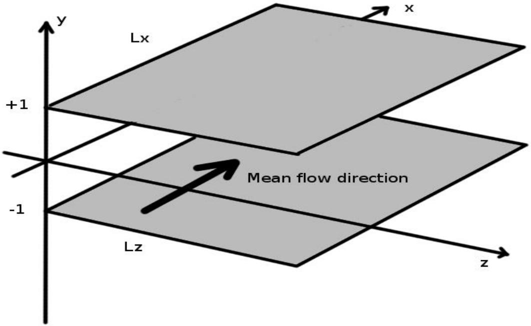
Outline

Drag reduction



A model problem: turbulent channel flow

Incompressible flow between two plane, parallel, infinite walls



- Flow is spatially invariant with x and z
- In DNS, tipically, $\approx 10^8$ d.o.f.s

Linear(ized) model

- Governing Navier–Stokes equations written in v–η formulation
- Linearization about a reference profile (direct approach)
- Fourier expansion in homogeneous directions: wavenumbers decouple

Recent evidence exists (e.g. Kim & Lim, PoF 2000; Kim, PoF 2003) that linear models are good for control design.

State-space form

Linear time-invariant (LTI) system:

$$\dot{x} = Ax + Bu + r$$
$$y = Cx + d$$

- Actuation: distributed wall blowing/suction at the walls
- Sensing: distributed skin friction or pressure measurements
- Nonlinearity and modeling errors recovered as a state noise (typically white)

Model-based optimal compensator

Goal: design of a feedback compensator minimizing

$$J = E\{x^H Q x + u^H R u\}$$

- Separation theorem: optimal controller and state estimator can be designed separately
- Requires the solution of two matrix Riccati equations having the same dimension of A

Open issues

- A state-space realization is required from measured models
- Accounting for the full space-time structure of the state noise is impractical
- Riccati-based design is prohibitively expensive for high-dimensional systems

A measured linear model

Looking for the average impulse response to wall forcing

- Wall forcing with a small space-time white Gaussian noise on the wall-normal velocity at the wall v_w
- In the linear setting, the perturbed flow reads:

$$v_{tot}(x, y, z, t) = \overline{v}(x, y, z, t) + v(x, y, z, t)$$
$$\eta_{tot}(x, y, z, t) = \overline{\eta}(x, y, z, t) + \eta(x, y, z, t)$$

A measured linear model

Computing the cross-correlation between the state and the wall forcing:

$$E\{v_{tot}(x'+x,y,z'+z,t'+t)v_w^*(x',z',t')\} = \dots$$
$$\dots \underbrace{E\{\overline{v}(x'+x,y,z'+z,t'+t)v_w^*(x',z',t')\}}_{=0} + \dots$$
$$\dots + E\{v(x'+x,y,z'+z,t'+t)v_w^*(x',z',t')\}$$

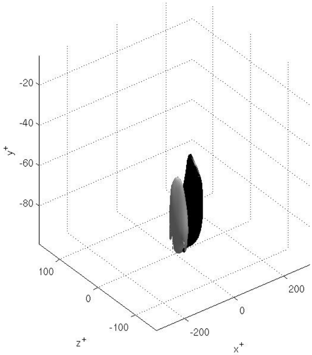
The average impulse response

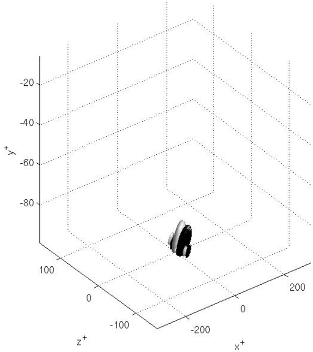
Leveraging a result in linear system theory, the state-forcing cross-correlation defines the impulse response of a LTI system:

$$H_v(x,y,z,t) = E\{v(x'+x,y,z'+z,t'+t)v_w^*(x',z',t')\}$$
$$H_\eta(x,y,z,t) = E\{\eta(x'+x,y,z'+z,t'+t)v_w^*(x',z',t')\}.$$

This function represents the average response of a turbulent channel flow when impulsive wall forcing on v is applied.

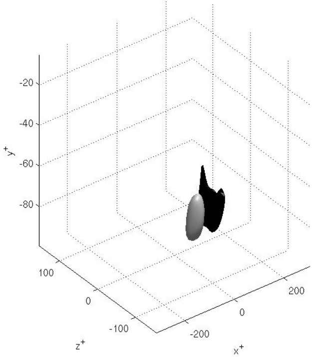
The average impulse response

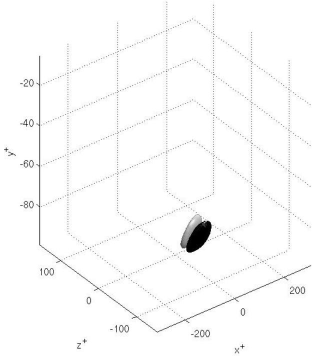




$H_v(x, y, z, t^+ = 5)$ $H_\eta(x, y, z, t^+ = 5)$

The average impulse response

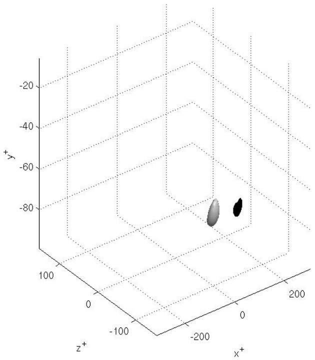




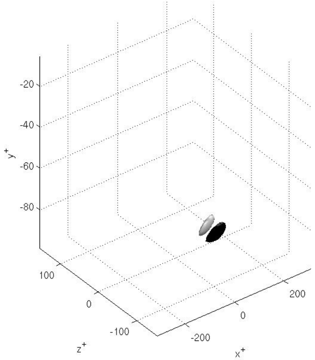
$H_v(x, y, z, t^+ = 15)$

$H_\eta(x, y, z, t^+ = 15)$

The average impulse response

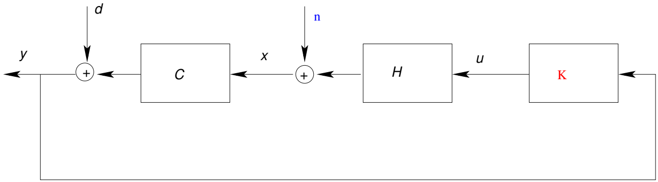


$H_v(x, y, z, t^+ = 25)$



$H_\eta(x, y, z, t^+ = 25)$

Feedback control loop



- n represents turbulent fluctuations in the uncontrolled flow
- The aim is to design a K such that the expectation

$$J = E\{x^H Qx + u^H Ru\}$$

is minimized in the closed loop system

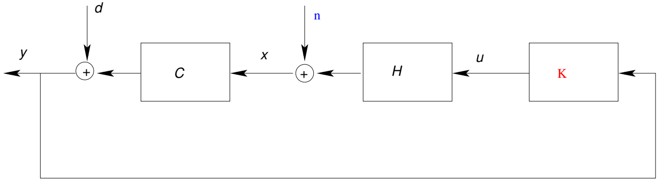
Compensator design

- It would be nice to avoid a state-space realization of H
- It would be very nice to devise a procedure reducing the complexity of the compensator design

However…

- In this problem, LTI systems with wide-sense stationary stochastic forcing are considered
- A frequency domain approach is feasible

Frequency domain approach



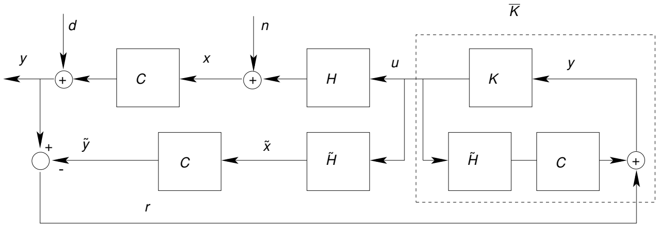
Rewriting the objective functional in frequency:

$$J = \int_{-\infty}^{+\infty} Tr[Q\phi_{xx}(f)] + Tr[R\phi_{uu}(f)]\ df.$$

Substituting, J is not quadratic in K.

Internal Model Control structure

Introducing a model $\tilde{H}$ of the plant H:



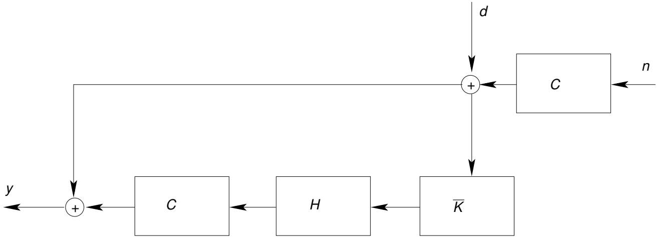
and $\overline{K}$ may be written as:

$$\overline{K} = (I - KC\tilde{H})^{-1}K.$$

This change of variable is known as Youla's parametrization (Morari & Zafiriou, 1998).

Equivalent open loop system

It is assumed that $\tilde{H} = H$ (modeling errors included in n only).



Substituting, J is now quadratic in $\overline{K}$!

Optimal compensator in frequency domain

$$J = \int_{-\infty}^{+\infty} Tr\Big\{ Q\phi_{nn} + QH\overline{K}C\phi_{nn} + Q\phi_{nn}C^H\overline{K}^H H^H + \dots$$

$$\dots + QH\overline{K}C\phi_{nn}C^H\overline{K}^H H^H + QH\overline{K}\phi_{dd}\overline{K}^H H^H \Big\} + \dots$$

$$\dots + Tr\Big\{ R\overline{K}C\phi_{nn}C^H\overline{K}^H + R\overline{K}\phi_{dd}\overline{K}^H \Big\} \, df.$$

- Minimization leads to the best possible LTI compensator for the problem at hand
- However, such compensator is noncausal

Causality enforcement

Introducing an appropriate Lagrange multiplier to enforce causality

$$J = \int_{-\infty}^{+\infty} Tr\Big\{Q\phi_{nn} + QH\overline{K}_{+}C\phi_{nn} + Q\phi_{nn}C^H\overline{K}_{+}^H H^H \dots$$

$$\dots + QH\overline{K}_{+}C\phi_{nn}C^H\overline{K}_{+}^H H^H + QH\overline{K}_{+}\phi_{dd}\overline{K}_{+}^H H^H\Big\} + \dots$$

$$\dots + Tr\Big\{R\overline{K}_{+}C\phi_{nn}C^H\overline{K}_{+}^H + R\overline{K}_{+}\phi_{dd}\overline{K}_{+}^H\Big\} + Tr[\Lambda_{-}\overline{K}_{+}^H]\, df.$$

Plus and minus subscripts denote frequency response functions of causal and anticausal response functions, respectively.

Wiener-Hopf problem

Minimization leads to the following Wiener-Hopf problem:

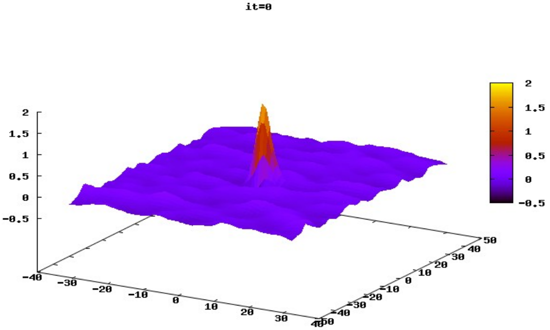
$$(H^H Q H + R)\overline{K}_+(C\phi_{nn}C^H + \phi_{dd}) + \Lambda_- = -H^H Q\phi_{nn}C^H$$

- Solution to this problem yields directly the compensator's frequency response, without invoking the separation theorem
- Directly accounts for the input-output relation (no model order reduction needed)
- Noise spectral densities apper in functional form $C\phi_{nn}$: full space-time structure of the noise easily accounted for
- Scalar equation for the single-input/single-output case

Compensator design and testing procedure

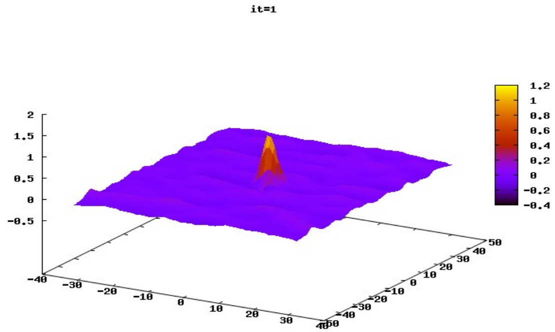
- Response function and noise spectral densities are measured via DNS and Fourier transformed in x and z
- Wiener-Hopf problem is solved wavenumber-wise
- Compensators are tested in a full nonlinear DNS

Compensator kernel in physical space



$$u(x,z,t) = \int K(x-x', z-z', t-t')y(x',z',t')\,dx'\,dz'\,dt'$$

Compensator kernel in physical space



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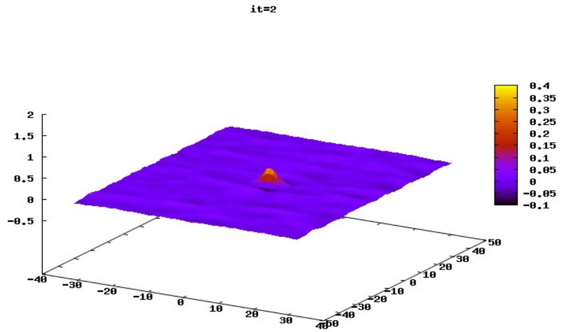
Compensator kernel in physical space



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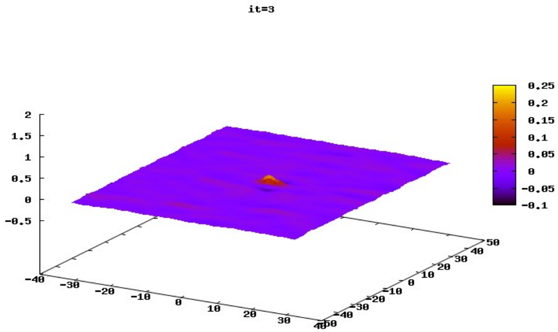
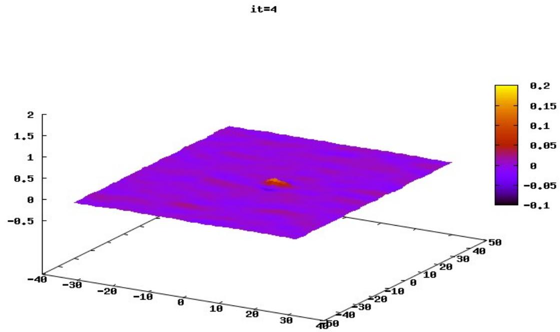


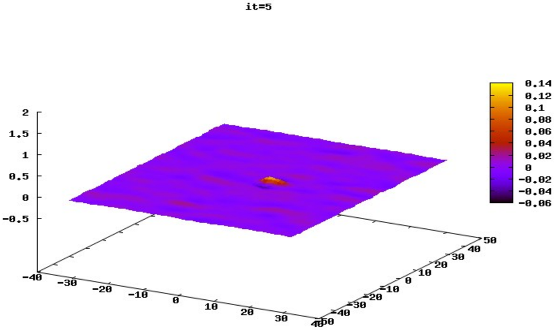
$$u(x,z,t) = \int K(x-x', z-z', t-t') y(x', z', t')\, dx'\, dz'\, dt'$$

Compensator kernel in physical space



$$u(x,z,t) = \int K(x-x', z-z', t-t') y(x', z', t')\, dx'\, dz'\, dt'$$

Compensator kernel in physical space



$$u(x,z,t) = \int K(x-x', z-z', t-t') y(x', z', t') \, dx' dz' dt'$$

Performance assessment

Parametric study addressing:

- Objective functional (Q matrix based on energy and dissipation)
- Actuation/sensing technique
- *Re* effects

More than 300 DNS ($\approx$ 40 years of CPU time) run at the supercomputing system located at the University of Salerno.

Best performance results

	Dissipation			Energy		
Re_τ	τ_x	τ_z	p	τ_x	τ_z	p
100	2%	0%	0%	0%	0%	0%
180	8%	6%	0%	0%	0%	0%

- Energy norm is ineffective
- Dissipation norm is effective
- Pressure measurement alone is not useful
- “Inverse” *Re*-effect when using dissipation norm

Overall best performance with $\approx 7.7\%$ of net power saved.

“Inverse” *Re*-effect

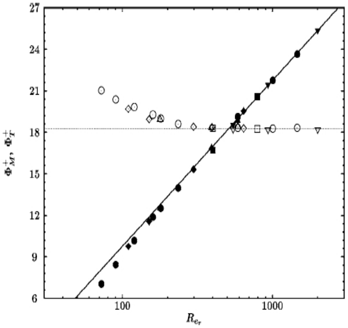
The performance of dissipation-based compensators improves with *Re*. Why?

$$\left.\frac{d\langle U\rangle}{dy}\right|_w = -\frac{1}{U_B}\left\langle \underbrace{\sum_{(\alpha,\beta)\neq(0,0)} D(\alpha,\beta)}_{D_{turb}} + \underbrace{\frac{1}{2}\int_{-1}^{1}\left(\frac{\partial \hat{U}}{\partial y}\right)_{(0,0)}\left(\frac{\partial \hat{U}}{\partial y}\right)^{*}_{(0,0)} dy}_{D_{mean}} \right\rangle$$

- D_{turb} is affected directly by zero net mass flux blowing/suction
- D_{mean} is affected indirectly via nonlinear interactions between fluctuations and the mean flow

“Inverse” *Re*-effect

But the relative contribution of the D_{turb} to the total dissipation increases with *Re*!



Re_τ	D_{turb}	D_{mean}
100	26.8%	73.2%
180	39.5%	60.5%

Laadhari, PoF 2007

Critical discussion

The use of linear estimators

- Present compensators incorporate Wiener filters (instead of Kalman filters) accounting for the full space-time structure of the state noise
- They are the best possible LTI filters for this problem
- However, their estimation capability is similar to that of Kalman filters
- This suggests that the use of linear filter is the issue

Substantial improvement may be obtained by using nonlinear filters, providing accurate estimates of the state far away from the wall.

Critical discussion

Selecting of appropriate cost functions

- Present compensators are the best possible LTI compensators for the problem at hand
- Their performance is, however, rather poor if compared to that of optimal compensators designed with analogous, state-space techniques

In the linear setting, the sole remaining degree of freedom is the cost function to be minimized.

Conclusions

- A novel cost-effective compensator design formulation has been proposed
- A measured linear model of the turbulent channel flow has been employed
- The approach accounts for the full time-space structure of the state noise

Possible developments

Control of wall turbulence

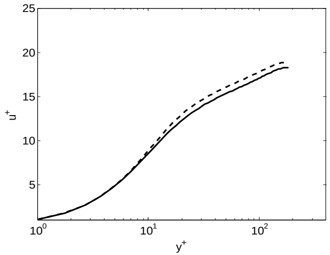
- Exploiting the compensator design methodology to optimize the cost function, with approximate models for the system dynamics and state noise
- Design based on experimentally measured shear-fluctuations cross-correlations
- Resorting to a state-space to optimize nonlinear (possibly reduced-order) estimators to be used in conjunction with standard optimal controllers

Possible developments

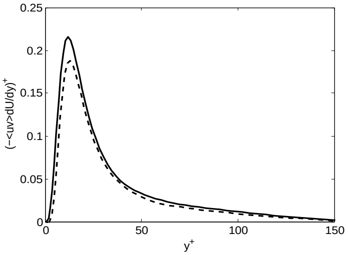
Design methodology

- Robust formulation in the IMC framework
- Multiple-input/multiple-output design
- Use of the present technique in a nonlinear optimization

Statistics of the controlled flow

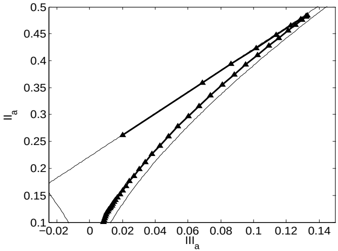


Mean velocity profile in the law-of-the-wall form

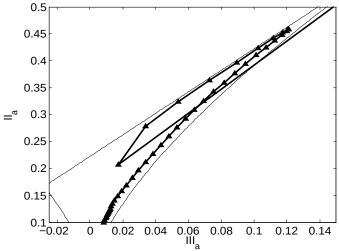


Production of turbulent kinetic energy

Anisotropy pattern

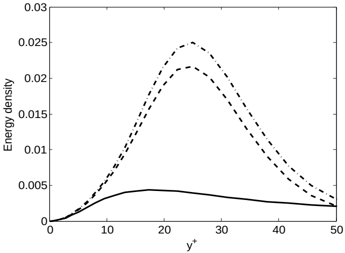


Uncontrolled case

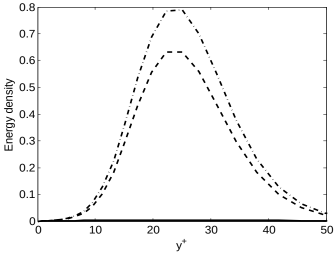


Controlled case

Wiener filter performance



Streamwise skin friction



Pressure